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SEMI-BLIND SUBSPACE IDENTIFICATION FOR MULTIPLE ANTENNA SYSTEM EQUALIZATION

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ABSTRACT: In this paper,we investigate the application of the subspace system identification (SSI) method (e.g. N4SID) to the MIMO frequency-selective fading channel estimation problem. The FIR constraint on the MIMO channel model is suggested to be relieved to draw benefit from possible parsimonious parametrization of the MIMO channel when subchannels become correlated. Also, the criterion for training sequence selection for SSIbased MIMO channel estimation is analyzed. Considering that the formalism of optimal input design is inappropriate for training sequence solution, we suggest still to use the conventional white and spatially uncorrelated sequences for SSI-based (non-FIR) MIMO channel estimation, even if they might be suboptimal. A modification of the SSI methods and a semi-blind approach are proposed to address the issue that only non-contiguous block-wise training sequences are available in practical mobile communication systems.

***Keywords*:** MIMO; Channel estimation; Subspace identification; Training sequences.

INTRODUCTION

Digital communication using multiple transmit and receive antennas has been one of the most important technical developments in modern communications. In a rich scattering environment, MIMO systems offer significant capacity gain at no cost of extra spectrum (Foschini and Gans, 1998). So far, most of the proposed MIMO transmission schemes assume channel state information (CSI) is known at the receiver. Therefore, the channel model needs to be identified at the receiver end. The most commonly used model for frequency-selective fading channels is a finite impulse response (FIR) model. FIR models for MIMO frequency-selective fading channels can be very non parsimonious since the number of parameters (tap gains) to be estimated in a FIR MIMO model increases rapidly with the number of transmit and receive antennas. For a FIR MIMO model with m transmit antennas and p receive antennas, a total number of $m\times p\times L$ parameters have to be estimated, where L is the lengthof the subchannels assuming all the subchannels have equal length. The FIR Model for a MIMO channel is not reducible when the subchannels are assumed to be independent, which can be justified in cases for which antennas are separated from each other by some multiple (e.g. 1/4) of the wavelength in both transmitting and receiving ends. However, when a large number of antennas are packed into a limited volume of space, the subchannels become correlated with each other (Chiurtu , 2001; Shiu , 2000). Hence the FIR model might be reduced to a more parsimonious state-space model.

Compared to the channel estimation methods based on FIR models of MIMO wireless channels, subspace system identification (SSI) methods (Van Overschee and De Moor, 1996; Verhaegen, 1994; Viberg, 1995), which are based on state-space modelling of the channel, could allow more parsimonious description of the MIMO channel or channel inverse if the subchannels share commonality to some extent. SSI algorithms identify the state-space model in a straightforward way and are numerically robust because they are based on computational tools such as singular value decomposition (SVD) and QR factorization. For MIMO systems with a relatively large number of transmit and receive antennas, the number of parameters to be estimated in the SSI method could be much less than that in methods based on FIR MIMO model. For time-varying single-input single-output (SISO) wireless channels, training sequence based methods have been widely used to estimate the channel explicitly or implicitly. A pre-selected sequence, known to both the transmitter and the receiver ahead of time, is transmitted through the channel and is captured by the receiver, where it is applied to adjust the adaptive equalizer in accordance with some optimization criterion, e.g. LMS algorithm. Other than trainingbased approaches, blind channel estimation has recently emerged as a promising technique for channel equalization because no training sequence is needed for this type of approaches (Tong , 1991). Instead, the knowledge is used that the transmitted symbols are distributed in a known way over a finite alphabet of fixed characters. However, most blind estimation methods suffer from convergence problem and have not found wide application in mobile communication with rapidly varying channels. This paper focuses on the discussion on the application of subspace system identification methods to the training-based MIMO channel estimation problem. In system identification terms, the presence of a training signal corresponds toknowledge of the input signal to the system (here the channel) being identified. For broadband FIR MIMO channel with subchannels being independent of each other, the optimal training sequences that achieve the minimum mean square error (MMSE) of channel estimation have an impulselike auto-correlation sequence and zero cross correlation (Fragouli , 2003). However, for a non- FIR MIMO channel with correlated subchannels, it is not clear that white and uncorrelated sequences are "optimal" for channel identification. In fact, the "optimal" choice should depend on the knowledge of the specific channel (Goodwin and Payne, 1977), which implies that the training sequence should adapt to the change of the channel. Considering the high complexity of designing "optimal" training sequence for non-FIR MIMO channel and the fact that training sequences in wireless communication systems are normally selected ahead of time and stay fixed during the transmission, we believe white and uncorrelated training sequences are still the best option. Another issue with the SSI-based MIMO channel estimation is that the training sequences may not be contiguous in the data stream in practical mobile communication systems. Instead, they appear as the mid-amble of a frame of data. More specifically, in the Groupe Speciale Mobile (GSM) system, a 26-bit long segment in the middle of each 156-bit frame is allocated for the insertion of the training sequence (Steele, 1992). A semi-blind approach can be efficient since it utilizes both the known data (training sequences) and unknown data (information sequences) to estimate the channel. Also, since the traditional SSI methods assume the availability of a contiguous input-output data stream, they need to be modified to suit the situation of MIMO channel estimation. It will be shown that when the length of the training sequence, Nt , is sufficiently large compared to the order or the McMillan degree of the model of the MIMO channel, the modified non-contiguous-data approach retains similar performance to the original contiguousdata approach.

The remainder of the paper is organized as follows. Section 2 overviews subspace system identification with application to MIMO channel estimation. The difference between subspace system identification methods and signal subspace methods that has been used in blind channel estimation is explained. Section 3 discusses the design of training sequences for subspace identification of MIMO frequency-selective fading channels. The formulation of SSI methods for non-contiguous data streams is discussed in Section 4. The conclusion follows in Section 5.

2. SUBSPACE SYSTEM IDENTIFICATION AND MIMO CHANNEL ESTIMATION

2.1 Channel Model

Single-input single-output (SISO) frequency-selective fading channels have been commonly modelled as tapped delay lines to characterize the multipath fading phenomenon. For a SISO FIR channel, the number of channel parameters to be estimated is equal to the length of the impulse response L. This parametrization could be very non-parsimonious for a broadband MIMO channel, which would contain $m\times p\times L$ unknown parameters assuming all the subchannels have equal length L, where m and p are the number of transmit and receive antennas, respectively. In the case where the subchannels in the MIMO system share commonality to some extent, a statespace model may be able to provide a more parsimonious parametrization of the frequency-selective fading channel than FIR model. Consider a system that employs m transmit and p receive antennas.

$$Xk+1=Axk+Buk$$
$$Yk=Cxk+Duk+nk \quad (1)$$

where $\mathbf{u}k$ is a $m\times 1$ vector that represents the channel input (symbols sent by the m transmit antennas) at time k. $\mathbf{y}k$ is a $p\times 1$ vector that represents the channel output at time k, i.e. the received symbols by the p receive antennas. $\mathbf{x}k$ is the q channel state vector where q is the order or the McMillan degree of the MIMO system. Additive white Gaussian noise is assumed and is represented by $\mathbf{n}k$. A, B, C and D are the system matrices in the state-variable description of the MIMO channel with obvious dimensions. If the impulse responses of the subchannels are correlated with each other to some extent, the system order q could be much less than the length of the impulse response of a single subchannel. Therefore the statevariable methods could allow a dramatic reduction in the number of parameters to be estimated for the MIMO equalizer compared to the case with a MIMO FIR model. For example, a 4×4 10-tap FIR channel model without other structure would require 160 parameters to be estimated. A state-variable realization with q poles would require no more than 169 parameters for q = 9 and 81 parameters for q = 5.

2.2 Subspace System Identification

Subspace system identification (SSI) refers to a class of recent algorithms, such as N4SID and MOESP, which apply input-output system identification methods to determine directly a state-space realization of system. The key idea of SSI methods is

to estimate the extended observability matrix through projection of future input-output data onto past input-output data. Then the system matrices *A*, *B*, *C* and *D* are computed based on the estimated observability matrix and singular value decomposition (SVD) algorithm. Refer to (Van Overschee and De Moor, 1996; Ljung, 1999) for details about SSI algorithms. Based on the channel model given in (1), SSI methods require the input to satisfy the following requirements for the channel to be identifiable.
(1) The input $\mathbf{u}k$ is uncorrelated with the additive Gaussian white noise nk.
(2) The input $\mathbf{u}k$ is persistently exciting of order of at least 2 times the maximum order of the channel.
(3) The symbols in the input sequence are contiguous and for consistency the number of input goes to infinity.

The first assumption is usually satisfied for wireless communication systems. The second one requires the training sequence to maintain a certain structure. Also, notice that the third assumption places limitation on the application of SSI methods to channel estimation in wireless communication systems where the training sequences are usually not contiguous in time. Instead, they lie in the mid-amble of a frame and are separated by data symbols, the knowledge of which is not shared between transmitter and receiver. This fact may suggest the use of recursive version of SSI methods. The issue of training sequence design for MIMO channel estimation will be addressed in detail later in Section 3 and 4.

2.3 Subspace-based MIMO Channel Estimation

We should point out that despite the similar name, the SSI-based methods differ from "Signal Subspace Methods" for blind MIMO channel estimation which seek to separate the noise and signal subspaces using singular value decomposition on the covariance matrix of the channel output (Moulines , 1995). In (Moulines , 1995), channel structure is constrained to be FIR with known input covariance. Moreover, the assumption that the channel matrix is block Toeplitz (FIR) is explored to estimate the channel up to a scale factor through singular value decomposition of the channel output covariance matrix. As for SSIbased methods, since the FIR constraint on the channel is relieved to draw benefit from possible parsimonious parametrization of the channel, the channel estimate cannot be obtained directly by applying SVD on the output covariance matrix. Instead, a more general approach is taken to estimate the extended observability matrix from SVD of the projection of input-output data, and then use the extended observability matrix to compute the channel estimate. There has been one attempt to use results from both signal subspace methods and SSI methods for blind channel estimation. In (Vandaele and Moonen, 2000), the approach of estimating extended observability matrix is taken under the assumption that the channel is FIR, i.e., matrix *A* in the state-variable model (1) has a fixed shifting matrix structure. In wireless communication systems, training sequences are usually placed at the mid-amble of data frames. It is fairly clear that SSI is feasible for a continuous stream of data. But it is less clear that a block-wise sequence of mid-ambles is possible to be used. There are some papers which discuss recursive subspace system identification such as (Lovera , 2000).

3. TRAINING SEQUENCE DESIGN FOR SSI-BASED MIMO CHANNEL ESTIMATION

Given that the channel can be estimated with the aid of off-line designed training sequences, the question arises as how to design optimal training sequences so that the error in the channel estimate can be reduced to the minimum. For an FIR MIMO channel with independent subchannels, it is believed that white and zero spatial crosscorrelation training sequences achieve the minimum mean square error (MMSE) of the estimates of channel coefficients (Caire and Mitra, 1998; Fragouli , 2003). However, for non-FIR MIMO channels, it is not clear that white and uncorrelated sequences are still the optimal choice. In fact, the optimal choice of the input for identifying a non-FIR channel should depend on the specific channel (Goodwin and Payne, 1977). Consider a general single-input single-output system

$$y_k=H(z)u_k+n_k \qquad (2)$$

where *{uk}* and *{yk}* are the input sequence and output sequence, respectively, and *{nk}* is zero-mean additive white Gaussian noise with variance σ 2. $H(z)$ is the transfer function of the channel which can be non-FIR. If the estimator is assumed to be efficient, so that the parameter covariance matrix achieves the Cramér-Rao lower bound, then a suitable criterion of optimality of the choice of training sequence would be

$$u_o=\operatorname{argmin}_u[-\operatorname{logdet}(M)]$$

subject to the input power constraint

$$\frac{1}{N}\sum_{k=1}^{N_t} u_k^2 = 1 \qquad (4)$$

where Nt is the number of available training symbols, i.e. the length of the training sequence. det(·) represents the determinant of a matrix. *M* is Fisher’s information matrix given by

$$M = E_{Y|\theta}\{(\frac{\partial logp(Y|\theta)}{\partial\theta})^T\left(\frac{\partial logp(Y|\theta)}{\partial\theta}\right)\} \quad (5)$$

where θ is the vector of parameters in $H(z)$ and σ^2. For the system given in (2),

$$M = \frac{1}{\Sigma}\sum_{k=1}^{N_t}(\frac{\partial H(z)}{\partial \theta}u_k)^T(\frac{\partial H(z)}{\partial \theta}u_k) + M_c\mathrm{c} \quad (6)$$

where Mc is a constant matrix which does not depend upon the choice of the input sequence **u**=$\{uk\}$. When the channel is FIR and causal, $H(z)$ has the form

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{L-1} z^{-(L-1)} \quad (7)$$

The cost function (3) is minimized when the training sequence $\{uk\}$ is white. However, for non-FIR channels (e.g. rational transfer function) the choice of optimal training sequence depends on the structure of the channel transfer function and the complexity of optimal input design is very high (Goodwin and Payne, 1977). It can be implied from the case of SISO channel that the optimal training sequence design for a general MIMO channel would require certain knowledge of the channel. Furthermore, for time-varying non-FIR channels the optimal training sequences vary in time as well. This suggests the use of iterative and adaptive schemes for the selection of training sequence in broadband MIMO wireless channel, which requires extra communication of channel state information from receiver to transmitter and communication of training sequence selection from the transmitter to the receiver. In current wireless communication systems, training sequences are normally selected ahead of time and stayed fixed during the transmission. It is clear that the complexity of the adaptive schemes is too high to be realistic for practical communication systems. Therefore, the formalism of optimal input design is inappropriate for training sequence selection. An appropriate choice of training sequences for general MIMO channels seems to be still using white and uncorrelated sequences as in MIMO FIR channel. The advantages of white and uncorrelated training sequences are listed as follows.

(1) *They satisfy the identifiability requirement of subspace system identification*. For SSI methods, such as N4SID, the input is assumed to be persistently exciting (Van Overschee and De Moor, 1996). It is easy to show that white and uncorrelated sequences are persistently exciting of any order.

(2) *They permit the use of simplified SSI algorithm for computing asymptotically unbiased matrices A, B, C and D*. In N4SID method, system matrices are computed based on a certain Kalman filter state sequence. The fact that this Kalman filter sequence cannot be calculated directly from data increases the complexity of the algorithm for computing asymptotically unbiased system matrices. However, if the input sequences are white, it is possible to use another *equivalent* Kalman filter sequence, which can be calculated directly from data, to compute the asymptotically unbiased system matrices, hence simplify the algorithm.

(3) *The performance improvement of optimal training sequence over white &uncorrelated might be small*. As shown in (Goodwin and Payne, 1977), for a typical SISO channel, the improvement in parameter variances achieved by use of the optimal input signal is about 1.49dB compared with the use of the pseudo-random binary signal. This 1.49dB improvement does not seem to be worth the effort made to compute the optimal training sequence iteratively.

Based on the above advantages, white and uncorrelated training sequences are still the best option for the purpose of general frequency-selective fading MIMO channel estimation using SSI methods.

4. SSI FORMULATION FOR NON-CONTIGUOUS DATA STREAMS

In practical mobile communication systems, the knowninput data sequences, or training sequences, may not be contiguous in the data stream. In GSM, as stated before, they appear as the 26-bit mid-amble of a 156- bit frame. One approach to tackling this problem is to treat the equalized data and its received version as "known" input-output data and to use them for SSI channel estimation in addition to the known training sequences. This idea is related to semi-blind adaptation where both training sequences and information data sequences are exploited to estimate the channel. However, this approach still requires a continuous sequence of frames of data, which is not the case for the TDMA-based GSM system where a singe user is assigned only a part of the 8 TDMA time slots. In this later circumstance, the state evolution of the received data must be restarted at the frame boundaries. This is at variance with the standard formulation of SSI. We next embark on an introductory foray into the development of a suitable modification of the SSI algorithms. Consider the evolution of two contiguous Nt-symbollong blocks of received data, with the first block commencing at time t and the second commencing immediately thereafter at $t + Nt$. Then we may write the blocked state equations as,

$$Y_{t,i,j} = \Gamma X_{t,j} + HU_{t,i,j}$$
$$Y_{t+N_t,i,j} = \Gamma X_{t+N_t,j} + HU_{t+N_t,i,j}$$

where Γ is the extended observability matrix, H is the system block Toeplitz matrix of Markov parameters, and

$$Y_{t,i,j} = \begin{bmatrix} y_t & y_{t+1} & \cdots & y_{t+j-1} \\ y_{t+1} & y_{t+2} & \cdots & y_{t+j} \\ \vdots & \vdots & & \vdots \\ y_{t+i-1} & y_{t+i} & \cdots & y_{t+i+j-2} \end{bmatrix},$$
$$X_{t,j} = \begin{bmatrix} x_t & x_{t+1} & \dots & x_{t+j-1} \end{bmatrix},$$
$$U_{t,i,j} = \begin{bmatrix} u_t & u_{t+1} & \cdots & u_{t+j-1} \\ u_{t+1} & u_{t+2} & \cdots & u_{t+j} \\ \vdots & \vdots & & \vdots \\ u_{t+i-1} & u_{t+i} & \cdots & u_{t+i+j-2} \end{bmatrix}.$$

In standard SSI approaches, these are combined to form a new matrix equation,

$$Y_{t,i,j+N_t} = \Gamma X_{t,j+N_t} + HU_{t,i,j+N_t}$$

This absorbs the data vectors into the Hankel structure of the new *U* and *Y* matrices. This adds further columns to the equation to be solved for the observability matrix Γ. Next consider the availability of discontinuous *Nt* - symbol-long blocks of received data with the first block commencing at time *t* and the second at some later time *t* +*M* with *M* > *Nt* . Then, we still achieve the relationship

$$Y_{t,i,j} = \Gamma X_{t,j} + HU_{t,i,j}$$
$$Y_{t,i,j+N_t} = \Gamma X_{t,j+N_t} + HU_{t,i,j+N_t}$$

but now the absorption of the data into individual Hankel matrices is no longer possible, because of the non-contiguity of the received data. We may, however, write an augmented equation composed from the above set.

$$\begin{bmatrix} Y_{t,i,j} \; Y_{t+M,i,j} \end{bmatrix} = \Gamma \begin{bmatrix} X_{t,j} \; X_{t,M,j} \end{bmatrix} + H[U_{t,i,j} \; U_{t+M,i,j}]$$

This set of equations to be solved for Γ is comparable to the contiguous-data set of equations. It has the same number of rows, *i*, and has $i-1$ fewer columns. When the length of the training sequence, *Nt*, is sufficiently large compared to the dimension of the generalized observability matrix, Γ (which depends on the state dimension of the model), then the non-contiguousdata approach is similar in its estimation power to the contiguous-data approach.

CONCULSION

In this paper, we suggest to relieve the FIR constraint on the model of MIMO frequency-selective channel to draw benefit from possible parsimonious parametrization of the channel, and to use subspace system identification (SSI) methods to tackle the channel estimation problem. Also, the selection of training sequences for SSI-based MIMO channel estimation is analyzed. The complexity of using optimal training sequences is found intimidating. Because conventional white and uncorrelated sequences satisfy the persistent excitation requirement of SSI methods and offer comparable performance to the optimal sequences, they are considered still the best choice for general (non-FIR) MIMO channel estimation. Furthermore, a modification of the SSI methods and a semi-blind approach are proposed to address the issue that only non-contiguous block-wise training sequences are available in practical mobile communication systems.

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